CAPACITY COMPARISON OF MIMO SPATIAL MULTIPLEXING-OFDM BETWEEN CHANNEL STATE INFORMATION RECEIVER (CSIR) AND CHANNEL STATE INFORMATION TRANSMITTER – RECEIVER (CSIT-R)

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Abstract

Multiple Input Multiple Output (MIMO) use multiple transmit antenna and multiple receive antenna which offer better system performance and capacity in multipath fading channel. A lot of MIMO techniques have been developed when channel state information is provided in receiver, but hardly for that provided in transmitter. When transmitter recognizes channel state, it can be used to increase MIMO system capacity. One of several techniques that use channel state information in transmitter to achieve maximum capacity is Singular Value Decomposition (SVD). On the other hand, Orthogonal Frequency Division Multiplexing (OFDM) is a popular method in wireless communication to overcome spread multipath effect in high data rate system.

This research describe analysis of factors which influence MIMO spatial multiplexing capacity combined with OFDM system. And influence of channel state information transmitter-receiver to MIMO-OFDM system capacity was analyzed as well. Calculation of the capacity was done using SVD method approach. Simulation was created refer to IEEE 802.11a standard and tested in fading rayleigh multipath channel with gaussian noise. The result indicate that when transmitter knows the channel state, its capacity increase become 4.6 b/s/Hz.

Keywords : Channel capacity, MIMO *Spatial Multoplexing*, OFDM, Channel state information transmitter, *SVD*, IEEE 802.11a.

1. Introduction

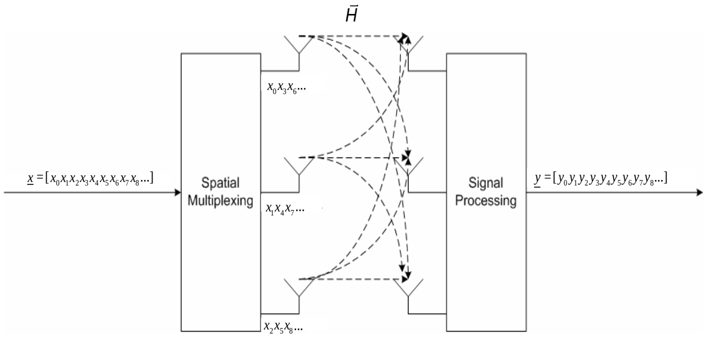
A lot of MIMO techniques have been developed and implemented on channel state information receiver (CSIR)[10], but rarely on channel state information transmitter-receiver (CSITR). In CSITR, the channel information could be used to increase system performance and capacity [13]. One of several techniques in CSITR is SVD method, which channel mattrix is decomposed to create independent channel with channel gain in each singular channel.

MIMO Scheme applied is spatial multiplexing. This research examine the influence of CSIT-R combined with OFDM to MIMO channel capacity and compare with CISR with compliance to IEEE 802.11a standard.

2. System Capacity of MIMO Spatial Multiplexing - OFDM

2.1 Spatial Multiplexing MIMO System

MIMO system use antenna diversity to achieve gain diversity. Gain multiplexing could be attained by means of spatial multiplexing or Space Division Multiplexing (SDM) to transmit signal. Basic principle of SDM is transmit symbol stream is divided into several parallel symbol stream and then transmitted simultaneously in the same bandwidth in each antenna. This technique could increase data rate.



Picture 1. MIMO with Spatial Multiplexing scheme

As depicted in picture 1, each substream transmitted simultaneously by transmit antenna will blend in air. In the receiver, a decoding technique is needed to attain each substream.

Channel matrix in MIMO channel (N- *transmit* antenna dan M- *receive* antenna) can be written as:

$$\vec{H} = \begin{bmatrix} h_{11} & h_{12} & \ldots\ldots & h_{1N} \\ h_{21} & h_{22} & \ldots\ldots & h_{2N} \\ . & & & \\ . & & & \\ h_{M1} & h_{M2} & \ldots\ldots & h_{M,N} \end{bmatrix} \qquad (1)$$

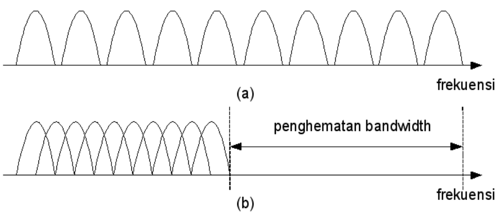
Where $h_{m,n}$ describe channel response of N transmit antenna to M receive antenna. If there is transmit signal $\underline{x} = [x_1\ x_2\ x_3\ \ldots.\ x_M]^T$, then y received signal is $\underline{y} = [y_1\ y_2\ y_3\ \ldots.\ y_N]^T$, where:

$$\underline{y} = \sqrt{\frac{E_s}{M}}\vec{H}\underline{x} + \underline{n} \qquad (2)$$

E_S is total average energy in transmitter during symbol period. Noise $\underline{n}$ is *zero mean circularly symmetry complex Gaussian* (ZMCSGC) noise.

2.2 OFDM

OFDM (*Orthogonal Frequency Division Multiplexing*) is multicarrier modulation technique with orthogonal subcarrier, so each subcarrier is overlapping and close spacing without intercarrier interference (ICI). This make OFDM system has spectrum efficiency better than conventional multicarrier modulation as depicted in picture 2 [4].



Picture 2. Multicarrier Spectrum (a) not overlap (b) Orthogonal Overlap

Concept of OFDM is to divide wideband information signal data rate into several parallel narrowband data rate and modulate it by orthogonal subcarrier.

Discrete Fourier Transform (DFT) is used in OFDM system to reduce complexity of transmitter and receiver system especially utilization of excessive oscillator, mixer and filter for each subcarrier. DFT is used to create orthogonal subcarrier and implemented by the Fast Fourier Transform (FFT) algorithm.

2.3 MIMO Channel Capacity

Baseband MIMO system with N transmit antenna and M received antenna could be modeled linier equation. Mutual information can be written by:

$$\begin{aligned} \mathrm{I}\,(\underline{x};\underline{y}) &= \mathrm{H}(\underline{y}) - \mathrm{H}(\underline{y} \mid \underline{x}) \\ &= \mathrm{H}(\underline{y}) - \mathrm{H}((\vec{\boldsymbol{H}}\underline{\boldsymbol{x}} + \underline{\boldsymbol{n}}) \mid \underline{x}) \\ &= \mathrm{H}(\underline{y}) - \mathrm{H}(\underline{\boldsymbol{n}} \mid \underline{x}) \\ &= \mathrm{H}(\underline{y}) - \mathrm{H}(\underline{\boldsymbol{n}}) \end{aligned} \qquad (3)$$

Where transmit vector $\underline{\boldsymbol{x}}$ and noise $\underline{\boldsymbol{n}}$ is assumed independent. Row 3 of equation (3) valid because ***H*** is assumed constant (null entropy) during transmit duration of whole $\underline{\boldsymbol{x}}$ vector. Equation (3) can be maximum when $\underline{\boldsymbol{y}}$ reach maximum entropy of $\mathbf{log_2det(}\boldsymbol{\pi e K)}$ with condition: circularly symmetric complex Gaussian $\underline{\boldsymbol{y}}$ vector with covariant matrix $E\{\underline{\boldsymbol{yy'}}\} = \boldsymbol{K}$. If transmit vector $\underline{\boldsymbol{x}}$ is also complex Gaussian, then ***K*** can be determined as follow:

$$\begin{aligned} \vec{K} &= E\left\{(\vec{\boldsymbol{H}}\underline{\boldsymbol{x}} + \underline{\boldsymbol{n}})(\vec{\boldsymbol{H}}\underline{\boldsymbol{x}} + \underline{\boldsymbol{n}})'\right\} \\ &= E\left\{\vec{\boldsymbol{H}}\underline{\boldsymbol{xx}}'\vec{\boldsymbol{H}}'\right\} + E\left\{\underline{\boldsymbol{nn}}'\right\} \\ &= \vec{\boldsymbol{H}}\vec{\boldsymbol{Q}}\vec{\boldsymbol{H}}' + \vec{K}^n \\ &= \vec{K}^s + \vec{K}^n \end{aligned} \qquad (4)$$

Where matrices $\boldsymbol{K^s}$ and $\boldsymbol{K^n}$ subsequently is part of signal and noise from covariant matrices ***K***. *Maximum mutual information* which describe channel capacity as well is:

$$\begin{aligned} C &= \mathrm{H}(\underline{y}) - \mathrm{H}(\underline{\boldsymbol{n}}) \\ &= \log_2[\det(\pi e(\vec{K}^s + \vec{K}^n)] - \log_2[\det(\pi e\vec{K}^n)] \\ &= \log_2[\det((\vec{K}^s + \vec{K}^n)(\vec{K}^n)^{-1})] \\ &= \log_2[\det((\vec{K}^s + \vec{K}^n)(\vec{K}^n)^{-1})] \\ &= \log_2[\det(\vec{\boldsymbol{H}}\vec{\boldsymbol{Q}}\vec{\boldsymbol{H}}'(\vec{K}^n)^{-1} + I_{n_R})] \end{aligned} \qquad (5)$$

where $\mathbf{I_{nx}}$ is identity matrix. Noise in each receive antenna is assumed uncorrelated, so $\mathbf{K^n{=}\sigma^2 I_{nx}}$, where $\boldsymbol{\sigma^2}$ is noise power in each received antenna. When transmitter doesn’t knows the channel, then optimal capacity can be reached by using the same power for each transmit antenna, $\mathbf{Q{=}(P_t/t)I_t}$, where $\mathbf{P_t}$ is total power of transmit signal. MIMO channel capacity can be written as follows:

$$C_{EP} = \log_2\left(\det\left[\mathbf{I}_M + \frac{\rho}{N}\vec{\mathbf{H}}\vec{\mathbf{H}}'\right]\right) \qquad (6)$$

Where $\boldsymbol{\rho = P_t/\sigma^2}$ is average *signal-to-noise ratio* (SNR) in the receiver (‘) indicate *transpose-conjugate*. $\vec{\boldsymbol{H}}$ is channel matrix M x N. Equation (6) refers to information source of each antenna which is Equal Power/EP Uncorrelated Source.

2.3.1 Unknown Channel in the Transmitter [5], [6]

For the unknown channel in transmitter, transmition capacity is indicated in equation (6). The equation can be written in form:

$$C = \log_2 \left\{ \left\{ \det \left(\underline{I}_M + \frac{E_s}{NN_0} \vec{H}\vec{H}^H \right) \right\} \right\} \quad (7)$$

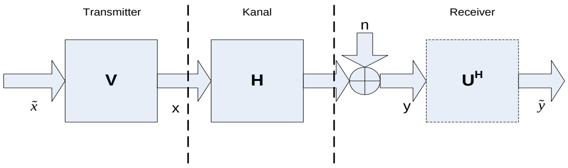
By doing decomposition process of channel matrix ***H*** using Singular Value Decomposition (SVD), it becomes:

$$C = \sum_{i=1}^{r} \log_2 \left(1 + \frac{E_s}{NN_0} \lambda_i \right) \quad (8)$$

Where $\boldsymbol{\lambda_i}$ (i=1,2,…,r) is eigen value of matrix $\boldsymbol{HH^H}$ and ***r*** is channel rank. From that equation, obviously that MIMO channel capacity is number of r channel SISO, which has power gain $\boldsymbol{\lambda_i}$ (i=1,2,…,r) and transmit power $\boldsymbol{E_s/N}$. Unknown channel in transmitter means for each independent signals has the same energy.

2.3.2 Know Channel in Transmitter [5], [6]

Scheme of decomposition if channel matrix ***H*** known by *transmitter* and *receiver* is :



Picture 3. CSIT-R channel decomposition scheme

Transmitter also could recognize channel condition if transmition scheme is TDD (*time division duplex*) or if there is feedback channel from receiver (*Closed-Loop Systems*).

Model signal as shown in equation (1) and and channel is decomposized by SVD, which is:

$$\vec{H} = \vec{U}\vec{D}\vec{V}^H \quad (9)$$

From the scheme above, before transmitted, signal or $\underline{\tilde{x}}$ vector with ZMCSGC properties is multiplied by matrix ***V*** so that:

$$\underline{x} = \vec{V}\underline{\tilde{x}} \quad (10)$$

In the receiver:

$$\underline{\tilde{y}} = \vec{U}^H \underline{y} \quad (11)$$

$$\underline{\tilde{y}} = \sqrt{\frac{E_s}{N}} \vec{U}^H \vec{H}\vec{V}\underline{\tilde{x}} + \vec{U}^H \underline{n} \quad (12)$$

By enclosing channel decomposition $\vec{H}$:

$$\underline{\tilde{y}} = \sqrt{\frac{E_s}{N}} \vec{U}^H \vec{U}\vec{D}\vec{V}\vec{V}^H \underline{\tilde{x}} + \vec{U}^H \underline{n}$$

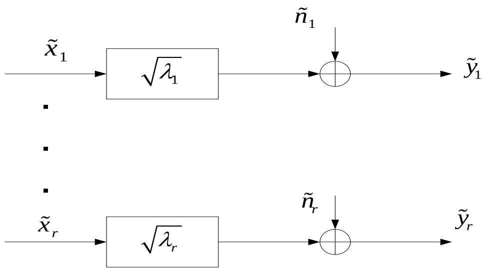
$$\underline{\tilde{y}} = \sqrt{\frac{E_s}{N}} \vec{D}\underline{\tilde{x}} + \underline{\tilde{n}} \quad (13)$$

Where D is singular value of channel i.

Equation (13) refers that if channel known by transmitter, then channel matrix ***H*** could be decomposed to be ***r*** SISO parallel channel which comply with:

$$\underline{\tilde{y}}_i = \sqrt{\frac{E_s}{N}} \sqrt{\lambda_i}\, \underline{\tilde{x}}_i + \underline{\tilde{n}}_i, \qquad i=1,2,\ldots,r \quad (14)$$

Its scheme can be shown as follow:



Picture 4. Channel scheme that is decomposed to become r SISO parallel channel

In case where transmitter knows the channel, power transmit is allocated to entire antenna element by means waterfilling power allocation method. It is possible because transmitter could acces sub channels to maximize channel capacity. MIMO channel capacity with waterfilling solution become [7]:

$$C_{WF} = \sum_{i=1}^{m} \log_2 \left(\mu \lambda_i \right)^+ \quad (15)$$

Where μ is chosen to fullfill :

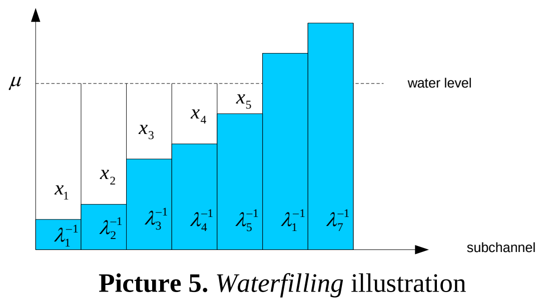
$$\rho = \sum_{i=1}^{r} \left(\mu - \lambda_i^{-1} \right)^+ \quad (16)$$

and "+" denote that only positif difference which have adding operation.

3.1 Simulated System Model

2.3.3 *Waterfilling* [7]

Waterfilling is a method used to allocate power so that, channel maximum capacity can be reached. It detect power distribution by using the height of water poured in a surface. Illustration of waterfilling method could be shown as follow:



Picture 5. *Waterfilling* illustration

In the picture above, $\boldsymbol{x_i}$ describe power distribution at subchannel i[th], so that:

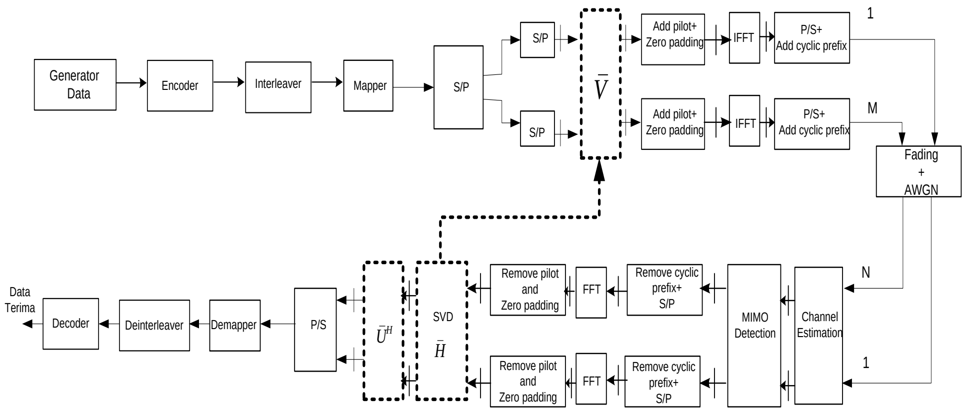
$$\sum_{i=1}^{k} x_i = P_T \quad (17)$$

$\boldsymbol{\lambda_i}$ is gain from subchannel i[th] and $\boldsymbol{\mu}$ is level of *waterfilling*. Power allocation from *waterfilling* method can be written as follow:

$$x_i = \left(\mu - \lambda_i^{-1} \right)^+ \quad (18)$$

where $(x_i)^+ = \max(0, x)$.

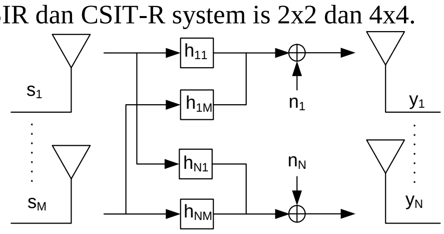
3. System Model and Simulation Parameter



Gambar 6. Modeling of MIMO Spatial Multiplexing-OFDM CSITR,
If without SVD then it becomes MIMO Spatial Multiplexing-OFDM CSIR

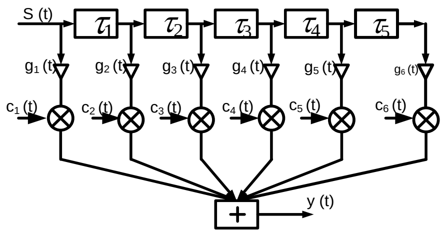
3.2 Fading Model Channel

MIMO channel model in this research refers to general equation of MIMO channel matrix as equation (1). Number of antenna combination used in CSIR dan CSIT-R system is 2x2 dan 4x4.



Picture 7. MIMO Channel Model

In the simulation, every ***h*** using *multipath fading Rayleigh* channel model with 6 *path delay*, as shown in next picture below:



Picture 8. *multipath fading* channel model

3.3 Simulation Parameters

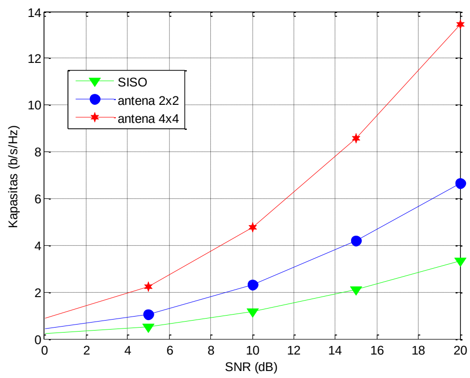
Tabel 1 Simulation Parameters

Parameter	Value/Type
Operation Frequency	5 GHz
OFDM Symbol number/Frame	20 OFDM symbol
Path Gain	[0 1 -1 -9 -10 -15 -20]dB
Path Delay	[0 310 710 1090 1730 2560] ns.
Length of *guard interval*	0.8 µs
OFDM symbol period	4 µs
Effective OFDM symbol period	3.2 µs
Carrier spacing	0.3125 MHz
Bandwidth nominal	20 MHz
Signal Mapper	QPSK
Interleaver & Deinterleaver	Matrix 16x120
Encoder& Decoder	*Convolutional*, Rate1/2
Polinomial	$[g_0\ \ g_1] = [133_8\ \ 171_8]$
Number of *subcarrier* data	48 *subcarrier*
Zero Padding	12
Number of *subcarrier pilot*	4
Point IFFT	64
User velocity	0, 2.7,43.2 km/hour
Doppler Frequency	0, 12.5, 200 Hz

4. Simulation Result

4.1 Influence of Number of Antenna

The difference between MIMO system and previous method (SISO) is in number of antenna usage, where in MIMO system, more than 1 antenna is used in transmitter or receiver. Picture 9 describe channel capacity for several number antenna in *multipath fading* Rayleigh channel. The comparison is in between SISO, MIMO-OFDM 2x2 and 4x4. all system is CSIR with still user (maximum Doppler frequency is 0 Hz).

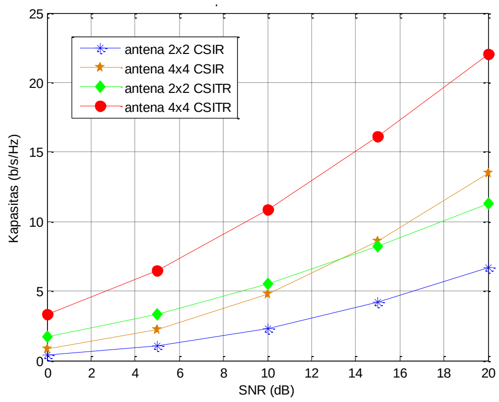


Picture 9. Capacity comparison of MIMO-OFDM CSIR channel in different number of antenna

Simulation graphic curve in picture 9 shown that 4x4 MIMO-OFDM configuration has channel capacity bigger than other configurations. The more antenna used the bigger channel capacity, because more parallel independent channel could happened. So, MIMO channel capacity is number of SISO channel capacity from decomposition process of channel matrix by SVD method.

4.2 Influence of CSIT-R on Channel Capacity

Picture 10 is a simulation graphic for channel capacity in CSIR and CSIT-R. Number of antenna used is 2x2 and 4x4 in Rayleigh multipath fading environment and maximum Doppler frequency 0 Hz.



Picture 10. MIMO-ODFM Channel Capacity Comparison between CSIR and CSIT-R

Picture 10 shows that MIMO-OFDM channel capacity with 4x4 or 2x2 antenna in CSIT-R is bigger than CSIR. Capacity for each antenna arrangement at 20dB SNR is compared in table 2 listed below.

Tabel 2. MIMO-OFDM Channel Capacity at SNR 20 dB

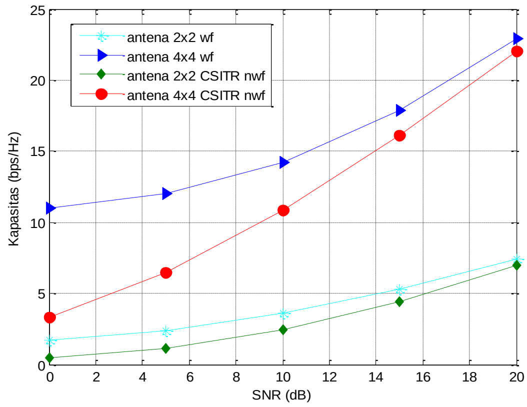
System Variation	Capacity (bps/Hz)	Capacity Improvement (bps/Hz)
2x2_CSIR	6.661194463	4.608153262
2x2_CSITR	11.26934773	
4x4_CSIR	13.46321314	8.560054192
4x4_CSITR	22.02326733	

It can be concluded that by using CSIT-R, system capacity can be increased. The improvement could reach twice bigger than CSIR. It is because there is a feedback to transmitter which inform it the channel status. The feedback comes from channel estimation that process training signal.

On the other side, altough MIMO-OFDM with CSIT-R could make bigger capacity, the system complexity is higher as well. In the implementation, to achieve optimum capacity, it has to be considered trade-off between complexity, performance, and system capacity, and number of antenna.

4.3 Influence of Power Allocation (*Waterfilling*) On Channel Capacity

There are two methods simulated in CSIT-R MIMO-OFDM system, which are: uniform power distribution and eigen value-based power distribution. Picture 11 indicate simulation result of CSIT-R MIMO-OFDM system capacity with uniform and waterfilling technique power distribution. Simulation is conducted in Rayleight multipath fading and fixed user environment.



Picture 11. Influence of Power Distribution on CSIT-R MIMO-OFDM Channel Capacity

Simulation result shown in picture 11 describe that waterfilling technique could increase channel capacity. It could be seen that in low SNR, channel capacity of waterfilling technique is bigger than it of uniform method. This means that if SNR that waterfilling usage is effective in negatif SNR (noise bigger than carrier).

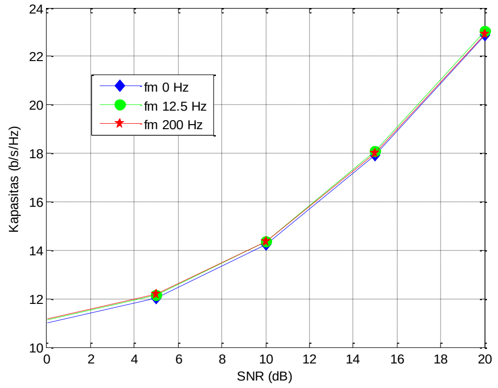
It is shown in that graphic that when SNR > 15dB , capacity of waterfilling method of 2x2 and 4x4 MIMO-OFDM is the same. It is also proof that waterfilling power distribution has a SNR limitation. In other word, waterfilling is very good if it is used on low SNR. In high SNR waterfilling method will have similar performance with uniform method.

Channel capacity improvement in 2x2 MIMO-OFDM configuration is not significant because the difference of gain in its sigular channel is not bigger than 4x4. So that the waterfilling method is only have slight influence on 2x2 CSIT-R MIMO-OFDM.

4.4 Influence of Doppler Effect on Channel Capacity

Picture 12 shows the capacity of 4x4 CSIT-R MIMO-OFDM in difference Doppler frequency. Simulation is conducted in Rayleigh multipath fading environment with doppler frequency of 0 Hz (static fading), 12.5 Hz (slow fading), and 200 Hz (slow fading).

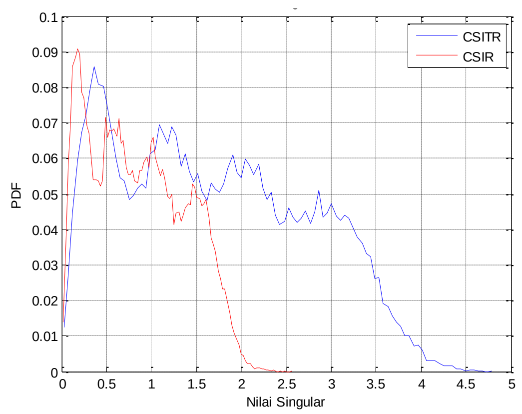
The graphic in picture 12 shows that the higher the Doppler frequency or the faster user movement, the system capacity does not experience significant difference. Theoritically, the higher the Doppler frequency, the more fluctuative the channel. It cause varying channel matrix elements in order to guarantee full rank condition. Because of system capacity is a function of its channel matrix, so the user movement does not influence the system capacity (means full rank condition is always achieved).



Picture 12. Influence of Doppler Effect on CSIT-R MIMO-OFDM Channel Capacity.

4.5 Influence of Channel Matrix Singular value on Channel Capacity

MIMO Channel singular value has a relationship with MIMO channel capacity. It is shown in equation (8), where decomposition process of Matrix $\boldsymbol{HH^H}$ will obtain eigen value of MIMO channel matrix $\boldsymbol{H}$. Eigen value is a quadrate of channel singular matrix value. Mathematically, MIMO channel capacity is proportional with MIMO singular matrix channel value.



Picture 13. Probability density function (pdf) of MIMO-OFDM singular matrix channel value

Table 3. Average of singular matrix channel value

System Variation	Singular Value			
	Ant.1	Ant.2	Ant.3	Ant.4
CSIR	1.5495	1.0382	0.6072	0.2197
CSIT-R	3.0864	2.0714	1.2178	0.4471

Table 3 is average of MIMO-OFDM singular matrix channel value as a result in each antenna. It is shown in table 3 that singular value of CSIT-R is bigger than singular value of CSIR. It means that capacity is depend on MIMO singular matrix channel value and can be concluded that singular value is a gain control.

5. Concluding Remarks

MIMO channel capacity is influence of many factors, such as number of antenna, and transmitter that knows channel status.

The improvement of capacity at SNR 20dB on MIMO-OFDM CSIT-R 4x4 could reaches 4.6 b/s/Hz better than MIMO-OFDM CSIR system.

If transmitter knows the channel status combined with waterfilling method then it will give optimum capacity on low SNR compared with uniform power distribution.

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